Errata for *Ocean Mixing*, August 11, 2022

1. Figure B.4: The right y-axis label should be $10^4 \beta \Phi_{\mathrm{vshear}}$.

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2. Sections B.5 to B.7: The last terms in the numerator and denominator of B.27 were reversed. The effect is minor, but it propagates through other equations in these sections, making it easier to restate the sections rather than commenting on changes.

B.5 Vertical Displacement

The two-dimensional spectrum of vertical displacement, $\Phi_\zeta(\beta, \omega)$, is obtained by multiplying the normalized wave function (B.15) by the energy spectrum,

$$\begin{aligned} \Phi_\zeta(\beta,\omega) &= \left(\frac{Z^2(\omega)}{N^2}\right) \Phi_E(\beta,\omega) \quad \left[\frac{\mathrm{m}^2}{\mathrm{m}^{-1}\mathrm{s}^{-1}}\right] \\ &= \frac{2 j_* b E_0}{\beta_*^2 + \beta^2} \left(\frac{E}{E_0}\right) \left(\frac{2}{\pi}\right) \left(\frac{f}{\omega}\right) \left(\frac{(\omega^2 - f^2)^{1/2}}{\omega^2}\right) \left(\frac{N^2}{N^2 - f^2}\right) \end{aligned} \tag{B.27}$$

The one-dimensional spectrum of displacement versus vertical wavenumber is obtained using

$$\int_f^N \frac{(\omega^2 - f^2)^{1/2}}{\omega^3} d\,\omega = -\frac{\sqrt{N^2 - f^2}}{2N^2} + \frac{\sec^{-1}(N/f)}{2f} \tag{B.28}$$

With the integral the product of the last four terms is

$$\frac{1}{\pi}\left[-\frac{1}{((N/f)^2 - 1)^{1/2}} + \frac{(N/f)^2}{(N/f)^2 - 1} \sec^{-1}(N/f)\right]$$

which is 0.499 when $N/f = 1000$ and 0.389 when $N/f = 5$.

Therefore,

$$\Phi_\zeta(\beta) = \int_f^N \Phi_\zeta(\beta,\omega)\, d\omega \approx \frac{j_* b E_0}{\beta_*^2 + \beta^2}\left(\frac{E}{E_0}\right) \quad \left[\frac{\mathrm{m}^2}{\mathrm{s}^{-1}}\right] \tag{B.29}$$

is accurate within 0.1% when $N/f = 1,000$ and is a 30% overestimate when $N/f = 5$.

The one-dimensional spectrum of displacement versus frequency is obtained by integrating (B.27) with respect to β. Using (B.8),

$$\Phi_\zeta(\omega) = \int_0^{\beta_u} \Phi_\zeta(\beta,\omega)\, d\beta \qquad (B.30)$$
$$= b^2 E_0 \left(\frac{E}{E_0}\right)\left(\frac{N_0}{N}\right)\left(\frac{2}{\pi}\right)\left(\frac{f}{\omega}\right)\left(\frac{(\omega^2-f^2)^{1/2}}{\omega^2}\right)\left(\frac{N^2}{N^2-f^2}\right)$$

Integrating again yields displacement variance. First, integrating (B.29),

$$\langle\zeta^2\rangle = \int_0^{\beta_u} \Phi_\zeta\, d\beta \quad [\mathrm{m}^2] \qquad \text{(B.31)}$$
$$= \frac{b^2 E_0}{2}\left(\frac{N_0}{N}\right)\left(\frac{E}{E_0}\right) = (7.3^2)\left(\frac{N_0}{N}\right)\left(\frac{E}{E_0}\right)$$

Integrating (B.30) produces the same expression multiplied by $N^2/(N^2 - f^2)$. This factor is 1.00 for $N/f = 1,000$ and 1.05 for $N/f = 5$.

For typical stratification and the GM energy level, rms displacements vary from ≈ 2 m in the shallow pycnocline to ≈ 20 m in the abyss (Fig. B.3). Potential energy is 1/4 of total energy

$$PE = \frac{N^2 < \zeta^2 >}{2} \approx \frac{b^2 N_0^2 E_0}{4}\left(\frac{N}{N_0}\right)\left(\frac{E}{E_0}\right) = \frac{E_{iw}}{4} \quad \left[\frac{\mathrm{J}}{\mathrm{kg}}\right] \qquad \text{(B.32)}$$

B.6 Vertical Velocity

The spectrum of vertical velocity is ω^2 times the displacement spectrum,

$$\Phi_w(\beta,\omega) = \omega^2 \Phi_\zeta(\beta,\omega) \quad \left[\frac{(\mathrm{m/s})^2}{\mathrm{m}^{-1}\mathrm{s}^{-1}}\right] \qquad \text{(B.33)}$$
$$= \frac{2 j_* b E_0}{\beta_*^2 + \beta^2}\left(\frac{E}{E_0}\right)\left(\frac{2}{\pi}\right)\left(\frac{f}{\omega}\right)(\omega^2 - f^2)^{1/2}\frac{N^2}{N^2-f^2}$$

The spectrum of vertical velocity versus vertical wavenumber is obtained using

$$\int_f^N \frac{(\omega^2-f^2)^{1/2}}{\omega} d\,\omega = (N^2-f^2)^{1/2} - f\sec^{-1}(N/f) \qquad \text{(B.34)}$$

to get

$$\Phi_w(\beta) = \int_f^N \Phi_w(\beta,\omega)\, d\omega \quad \left[\frac{(\mathrm{m/s})^2}{\mathrm{m}^{-1}}\right] \tag{B.35}$$
$$= \frac{4j_* b N_0 E_0 f_{30}}{\pi(\beta_*^2+\beta^2)} \left(\frac{E}{E_0}\right)\left(\frac{f}{f_{30}}\right)\left(\frac{N}{N_0}\right)\left[\frac{(N/f)}{((N/f)^2-1)^{1/2}} - \frac{(N/f)\sec^{-1}(N/f)}{(N/f)^2-1}\right]$$

The term in brackets is 1 when $N/f = 1,000$, gradually decreases to 0.9 when $N/f = 21$, and is 0.75 when $N/f = 5$.

The one-dimensional spectrum of vertical velocity versus frequency is obtained most easily by multiplying (B.30) by ω^2,

$$\Phi_w(\omega) = \frac{2b^2E_0}{\pi}\left(\frac{E}{E_0}\right)\left(\frac{N_0}{N}\right)\frac{(N/f)^2}{(N/f)^2-1}\left(\frac{f}{\omega}\right)(\omega^2 - f^2)^{1/2} \tag{B.36}$$

Integrating both one-dimensional spectra yields the variance

$$< w^2 > = \int_{\beta_*}^{\beta_u} \Phi_w(\beta)\, d\beta = \int_f^N \Phi_w(\omega)\, d\omega \quad \left[(\mathrm{m/s})^2\right] \tag{B.37}$$
$$= \frac{2b^2 N_0 E_0 f_{30}}{\pi}\left(\frac{E}{E_0}\right)\left(\frac{f}{f_{30}}\right)\left[\frac{(N/f)}{((N/f)^2-1)^{1/2}} - \frac{(N/f)\sec^{-1}(N/f)}{(N/f)^2-1}\right]$$

The term in brackets was estimated for (B.35) and is a 30% overestimate when $N/f = 5$. It is no greater than a 10% error when $N/f > 21$. When $f = f_{30}$ and $N = N_0$ $< w^2 > \approx (5.1 \times 10^{-3})^2$ $(\mathrm{m/s})^2$.

B.7 Vertical Strain

The two-dimensional strain spectrum is obtained by multiplying the two-dimensional displacement spectrum by β^2,

$$\Phi_{strain}(\beta,\omega) = \beta^2\Phi_\zeta(\beta,\omega) \quad \left[\frac{1}{\mathrm{m}^{-1}\mathrm{s}^{-1}}\right] \tag{B.38}$$
$$= 2j_* b E_0 \frac{\beta^2}{\beta_*^2+\beta^2}\left(\frac{E}{E_0}\right)\left(\frac{2}{\pi}\right)\left(\frac{N^2}{N^2-f^2}\right)\left(\frac{f}{\omega}\right)\left(\frac{(\omega^2-f^2)^{1/2}}{\omega^2}\right)$$

Integrating with respect to frequency gives the same last four terms as those in (B.27) whose product was approximated as 0.5 which is a 30% overestimate when $N/f = 5$. Therefore, the same approximation gives

$$\Phi_{strain}(\beta) \approx j_* b E_0 \frac{\beta^2}{\beta_*^2+\beta^2}\left(\frac{E}{E_0}\right) \tag{B.39}$$

The one-dimensional spectrum of strain versus frequency is obtained by integrating (B.28) with respect to β. using the integral approximation in (B.23),

$$\Phi_{strain}(\omega) \approx 2j_* b E_0 \beta_u \left(\frac{E}{E_0}\right)\left(\frac{2}{\pi}\right)\left(\frac{N^2}{N^2-f^2}\right)\left(\frac{f}{\omega}\right)\left(\frac{(\omega^2-f^2)^{1/2}}{\omega^2}\right) \tag{B.40}$$

Strain variance is obtained by integrating by integrating the one-dimensional spectra,

$$\begin{aligned} < strain^2 > &= \int_0^{\beta_u} \Phi_{strain}(\beta)\, d\beta = \int_f^N \Phi_{strain}(\omega) l, d\omega \qquad \text{(B.41)} \\ &= j_* b \beta_u E_0 \left(\frac{E}{E_0}\right) = (0.496)^2 \left(\frac{E}{E_0}\right) \quad \text{[dimensionless]} \end{aligned}$$